Math/Textbook Portfolio

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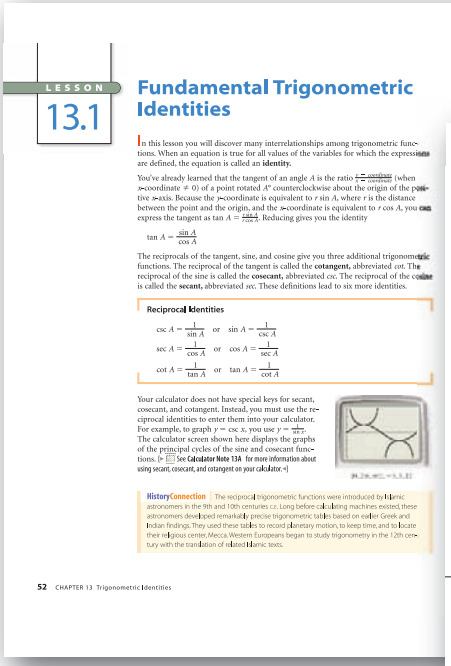


LESSON 13.1

Fundamental Trigonometric Identities

In this lesson you will discover many interrelationships among trigonometric functions. When an equation is true for all values of the variables for which the expressions are defined, the equation is called an **identity.**

You've already learned that the tangent of an angle A is the ratio $\frac{y}{x} = \frac{\text{opp/hyp}}{\text{adj/hyp}}$ (when x-coordinate $\neq 0$) of a point rotated $A°$ counterclockwise about the origin of the positive x-axis. Because the y-coordinate is equivalent to $r \sin A$, where r is the distance between the point and the origin, and the x-coordinate is equivalent to $r \cos A$, you can express the tangent as $\tan A = \frac{r \sin A}{r \cos A}$. Reducing gives you the identity

$$\tan A = \frac{\sin A}{\cos A}$$

The reciprocals of the tangent, sine, and cosine give you three additional trigonometric functions. The reciprocal of the tangent is called the **cotangent,** abbreviated *cot.* The reciprocal of the sine is called the **cosecant,** abbreviated *csc.* The reciprocal of the cosine is called the **secant,** abbreviated *sec.* These definitions lead to six more identities.

Reciprocal Identities

$$\csc A = \frac{1}{\sin A} \quad \text{or} \quad \sin A = \frac{1}{\csc A}$$
$$\sec A = \frac{1}{\cos A} \quad \text{or} \quad \cos A = \frac{1}{\sec A}$$
$$\cot A = \frac{1}{\tan A} \quad \text{or} \quad \tan A = \frac{1}{\cot A}$$

Your calculator does not have special keys for secant, cosecant, and cotangent. Instead, you must use the reciprocal identities to enter them into your calculator. For example, to graph $y = \csc x$, you use $y = \frac{1}{\sin x}$. The calculator screen shown here displays the graphs of the principal cycles of the sine and cosecant functions. (▶ 13 See **Calculator Note 13A** for more information about using secant, cosecant, and cotangent on your calculator.◀)

HistoryConnection The reciprocal trigonometric functions were introduced by Islamic astronomers in the 9th and 10th centuries C.E. Long before calculating machines existed, these astronomers developed remarkably precise trigonometric tables based on earlier Greek and Indian findings. They used these tables to record planetary motion, to keep time, and to locate their religious center, Mecca. Western Europeans began to study trigonometry in the 12th century with the translation of related Islamic texts.

Once you know a few identities, you can use them to prove other identities. One strategy for proving the truth of an identity is to verify that both sides of the equation are always equivalent. You can do this by working with one side of the equation until it is the same as the other side.

EXAMPLE A Prove this identity (assume $\sin A \neq 0$):

$$\cot A = \frac{\cos A}{\sin A}$$

▶ ***Solution*** Use definitions and identities that you know in order to show that both sides of the identity are equivalent. Be sure to work on only one side of the equation.

$\cot A \stackrel{?}{=} \frac{\cos A}{\sin A}$ The original identity.

$\frac{1}{\tan A} \stackrel{?}{=} \frac{\cos A}{\sin A}$ Use the reciprocal identity to replace $\cot A$ with $\frac{1}{\tan A}$.

$\frac{1}{\frac{\sin A}{\cos A}} \stackrel{?}{=} \frac{\cos A}{\sin A}$ Replace $\tan A$ with $\frac{\sin A}{\cos A}$.

$\frac{\cos A}{\sin A} = \frac{\cos A}{\sin A}$ $1 \div \frac{\sin A}{\cos A}$ is equivalent to $1 \cdot \frac{\cos A}{\sin A}$, or simply $\frac{\cos A}{\sin A}$.

Therefore, $\cot A = \frac{\cos A}{\sin A}$ is a true identity. To verify this, you can graph the two equations $y = \cot A$ and $y = \frac{\cos A}{\sin A}$ and verify that they give the same graph. Now that you have proven this identity, you can use it to prove other identities.

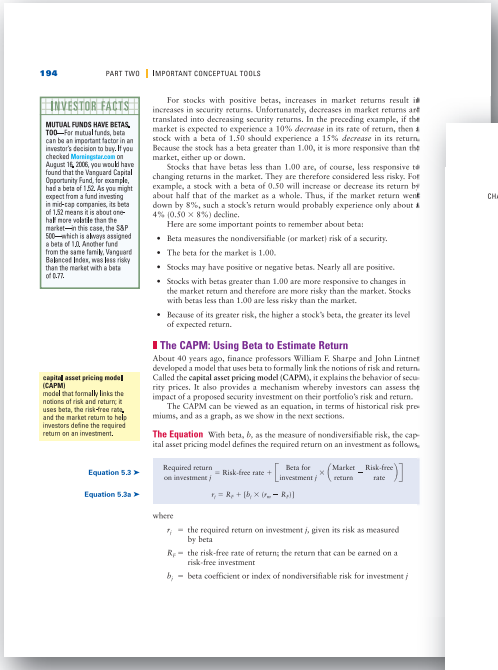
In the Investigation you will discover a set of trigonometric identities that collectively are called the *Pythagorean identities.* To prove a new identity, you are allowed to use any previously proven or accepted identity.

Investigation
Pythagorean Identities

Step 1 Use your calculator to graph the equation $y = \sin^2 x + \cos^2 x$. [You'll probably have to enter this as $y = (\sin x)^2 + (\cos x)^2$.] Use your graph to write an identity equation that is true for every value of x.

Step 2 Use the definitions of $\sin A$, $\cos A$, and the diagram shown here to prove your identity.

Investigating College Algebra and Trigonometry with Technology
Kathy Burgis
Key College Publishing (2006)



INVESTOR FACTS

MUTUAL FUNDS HAVE BETAS, TOO.—For mutual funds, beta can be an important factor in an investor's decision to buy. If you checked Morningstar.com on August 16, 2006, you would have found that the Vanguard Capital Opportunity Fund, for example, had a beta of 1.52. As you might expect from a fund investing in mid-cap companies, its beta of 1.52 means it is about one-half more volatile than the market—in this case, the S&P 500—which is always assigned a beta of 1.0. Another fund from the same family, Vanguard Balanced Index, was less risky than the market with a beta of 0.77.

For stocks with positive betas, increases in market returns result in increases in security returns. Unfortunately, decreases in market returns are translated into decreasing security returns. In the preceding example, if the market is expected to experience a 10% *decrease* in its rate of return, then a stock with a beta of 1.50 should experience a 15% *decrease* in its return. Because the stock has a beta greater than 1.00, it is more responsive than the market, either up or down.

Stocks that have betas less than 1.00 are, of course, less responsive to changing returns in the market. They are therefore considered less risky. For example, a stock with a beta of 0.50 will increase or decrease its return by about half that of the market as a whole. Thus, if the market return went down by 8%, such a stock's return would probably experience only about a 4% (0.50 × 8%) decline.

Here are some important points to remember about beta:

- Beta measures the nondiversifiable (or market) risk of a security.
- The beta for the market is 1.00.
- Stocks may have positive or negative betas. Nearly all are positive.
- Stocks with betas greater than 1.00 are more responsive to changes in the market return and therefore are more risky than the market. Stocks with betas less than 1.00 are less risky than the market.
- Because of its greater risk, the higher a stock's beta, the greater its level of expected return.

The CAPM: Using Beta to Estimate Return

capital asset pricing model (CAPM) model that formally links the notions of risk and return; it uses beta, the risk-free rate, and the market return to help investors define the required return on an investment.

About 40 years ago, finance professors William F. Sharpe and John Lintner developed a model that uses beta to formally link the notions of risk and return. Called the **capital asset pricing model** (CAPM), it explains the behavior of security prices. It also provides a mechanism whereby investors can assess the impact of a proposed security investment on their portfolio's risk and return.

The CAPM can be viewed as an equation, in terms of historical risk premiums, and as a graph, as we show in the next sections.

The Equation With beta, b_j as the measure of nondiversifiable risk, the capital asset pricing model defines the required return on an investment as follows:

Equation 5.3 ➤

$$\text{Required return on investment } j = \text{Risk-free rate} + \left[\text{Beta for investment } j \times \left(\text{Market return} - \text{Risk-free rate}\right)\right]$$

Equation 5.3a ➤

$$r_j = R_F + [b_j \times (r_m - R_F)]$$

where

r_j = the required return on investment j, given its risk as measured by beta

R_F = the risk-free rate of return; the return that can be earned on a risk-free investment

b_j = beta coefficient or index of nondiversifiable risk for investment j

r_m = the market return; the average return on all securities (typically measured by the average return on all securities in the Standard & Poor's 500-Stock Composite Index or some other broad stock market index)

The CAPM can be divided into two parts: (1) the risk-free rate of return, R_F, and (2) the *risk premium*, $b_j \times (r_m - R_F)$. The risk premium is the amount of return investors demand beyond the risk-free rate to compensate for the investment's nondiversifiable risk as measured by beta. The equation shows that *as beta increases, the risk premium increases, thereby causing the required returns for the given investment to increase.*

We can demonstrate use of the CAPM with the following example. Assume you are considering security Z with a beta (b_Z) of 1.25. The risk-free rate (R_F) is 6% and the market return (r_m) is 10%. Substituting these data into the CAPM equation, Equation 5.3a, we get:

$$r_z = 6\% + [1.25 \times (10\% - 6\%)] = 6\% + [1.25 \times 4\%]$$
$$= 6\% + 5\% = \underline{11\%}$$

You should therefore expect—indeed, require—an 11% return on this investment as compensation for the risk you have to assume, given the security's beta of 1.25.

If the beta were lower, say 1.00, the required return would be lower:

$$r_z = 6\% + [1.00 \times (10\% - 6\%)] = 6\% + 4\% = \underline{10\%}$$

If the beta were higher, say 1.50, the required return would be higher:

$$r_z = 6\% + [1.50 \times (10\% - 6\%)] = 6\% + 6\% = \underline{12\%}$$

Clearly, the CAPM reflects the positive mathematical relationship between risk and return: The higher the risk (beta), the higher the risk premium, and therefore the higher the required return.

Historical Risk Premiums Using the historical return data for selected security investments for the 1926–2005 period shown in Chapter 4, Table 4.4 on page 130, we can calculate the risk premiums for each investment category. The calculation (consistent with Equation 5.3) involves merely subtracting the historical U.S. Treasury bill's average return (the assumed risk-free rate of return, R_F) from the historical average return for a given investment:

Investment		Risk Premium*
Large-company stocks	12.3% − 3.8%	= 8.5%
Small-company stocks	17.4 − 3.8	= 13.6
Long-term corporate bonds	6.2 − 3.8	= 2.4
Long-term government bonds	5.8 − 3.8	= 2.0
U.S. Treasury bills	3.8 − 3.8	= 0.0

*Return values obtained from Chapter 4, Table 4.4 on page 130.

Fundamentals of Investing:International Edition
Lawrence Gitman
Addison-Wesley (2006)

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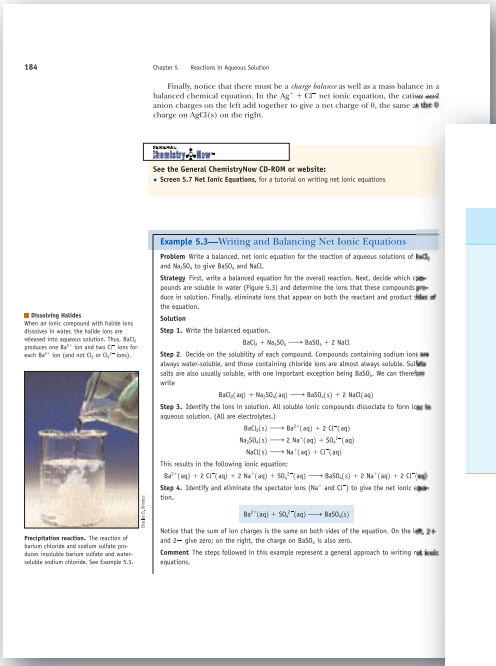
Introduction to Econometrics
Stock/Watson
Addison Wesley (2006)



184 Chapter 5 Reactions in Aqueous Solution

Finally, notice that there must be a *charge balance* as well as a mass balance in a balanced chemical equation. In the $Ag^+ + Cl^-$ net ionic equation, the cation and anion charges on the left add together to give a net charge of 0, the same as the 0 charge on AgCl(s) on the right.

See the General ChemistryNow CD-ROM or website:
- **Screen 5.7 Net Ionic Equations,** for a tutorial on writing net ionic equations

Example 5.3—Writing and Balancing Net Ionic Equations

Problem Write a balanced, net ionic equation for the reaction of aqueous solutions of $BaCl_2$ and Na_2SO_4 to give $BaSO_4$ and NaCl.

Strategy First, write a balanced equation for the overall reaction. Next, decide which compounds are soluble in water (Figure 5.3) and determine the ions that these compounds produce in solution. Finally, eliminate ions that appear on both the reactant and product sides of the equation.

Solution

Step 1. Write the balanced equation.

$$BaCl_2 + Na_2SO_4 \longrightarrow BaSO_4 + 2\ NaCl$$

Step 2. Decide on the solubility of each compound. Compounds containing sodium ions are always water-soluble, and those containing chloride ions are almost always soluble. Sulfate salts are also usually soluble, with one important exception being $BaSO_4$. We can therefore write

$$BaCl_2(aq) + Na_2SO_4(aq) \longrightarrow BaSO_4(s) + 2\ NaCl(aq)$$

Step 3. Identify the ions in solution. All soluble ionic compounds dissociate to form ions in aqueous solution. (All are electrolytes.)

$$BaCl_2(s) \longrightarrow Ba^{2+}(aq) + 2\ Cl^-(aq)$$
$$Na_2SO_4(s) \longrightarrow 2\ Na^+(aq) + SO_4^{2-}(aq)$$
$$NaCl(s) \longrightarrow Na^+(aq) + Cl^-(aq)$$

This results in the following ionic equation:

$$Ba^{2+}(aq) + 2\ Cl^-(aq) + 2\ Na^+(aq) + SO_4^{2-}(aq) \longrightarrow BaSO_4(s) + 2\ Na^+(aq) + 2\ Cl^-(aq)$$

Step 4. Identify and eliminate the spectator ions (Na^+ and Cl^-) to give the net ionic equation.

$$Ba^{2+}(aq) + SO_4^{2-}(aq) \longrightarrow BaSO_4(s)$$

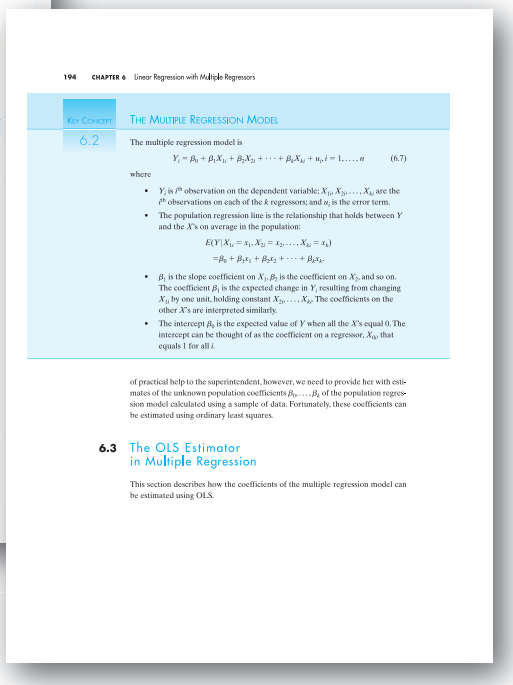
Notice that the sum of ion charges is the same on both sides of the equation. On the left, 2+ and 2− give zero; on the right, the charge on $BaSO_4$ is also zero.

Comment The steps followed in this example represent a general approach to writing net ionic equations.

Dissolving Halides
When an ionic compound with halide ions dissolves in water, the halide ions are released into aqueous solution. Thus, $BaCl_2$ produces one Ba^{2+} ion and two Cl^- ions for each Ba^{2+} ion (and not Cl_2 or Cl_2^{2-} ions).

Precipitation reaction. The reaction of barium chloride and sodium sulfate produces insoluble barium sulfate and water-soluble sodium chloride. See Example 5.3.

Chemistry & Chemical Reactivity
John C. Kotz
Thomson Brooks/Cole 2006



194 CHAPTER 6 Linear Regression with Multiple Regressors

KEY CONCEPT 6.2 THE MULTIPLE REGRESSION MODEL

The multiple regression model is

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \cdots + \beta_k X_{ki} + u_i,\ i = 1, \ldots, n \qquad (6.7)$$

where

- Y_i is i^{th} observation on the dependent variable; $X_{1i}, X_{2i}, \ldots, X_{ki}$ are the i^{th} observations on each of the k regressors; and u_i is the error term.
- The population regression line is the relationship that holds between Y and the X's on average in the population:

$$E(Y|X_{1i} = x_1, X_2 = x_2, \ldots, X_{ki} = x_k)$$
$$= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k.$$

- β_1 is the slope coefficient on X_1, β_2 is the coefficient on X_2, and so on. The coefficient β_1 is the expected change in Y_i resulting from changing X_{1i} by one unit, holding constant $X_{2i}, \ldots, X_{ki}$. The coefficients on the other X's are interpreted similarly.
- The intercept β_0 is the expected value of Y when all the X's equal 0. The intercept can be thought of as the coefficient on a regressor, X_{0i}, that equals 1 for all i.

of practical help to the superintendent, however, we need to provide her with estimates of the unknown population coefficients $\beta_0, \ldots, \beta_k$ of the population regression model calculated using a sample of data. Fortunately, these coefficients can be estimated using ordinary least squares.

6.3 The OLS Estimator in Multiple Regression

This section describes how the coefficients of the multiple regression model can be estimated using OLS.

$$\text{Required return on investment } j = \text{Risk-free rate} + \left[\text{Beta for investment } j \times \left(\text{Market return} - \text{Risk-free rate}\right)\right]$$

$$r_j = R_F + [b_j \times (r_m - R_F)]$$

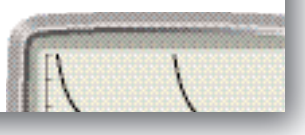
$+ b_k X_{ki}$, and the mistake in predicting Y_i is $Y_i - (b_0 + b_1 X_{1i} + \cdots + b_k X_{ki}) = Y_i - b_0 - b_1 X_{1i} - \cdots - b_k X_{ki}$. The sum of these squared prediction mistakes over all n observations thus is

$$\sum_{i=1}^{n} (Y_i - b_0 - b_1 X_{1i} - \cdots - b_k X_{ki})^2. \qquad (6.8)$$

$$\frac{\cos A}{\sin A} = \frac{\cos A}{\sin A}$$

$1 \div \frac{\sin A}{\cos A}$ is equivalent to $1 \cdot \frac{\cos A}{\sin A}$, or simply $\frac{\cos A}{\sin A}$.

Therefore, $\cot A = \frac{\cos A}{\sin A}$ is a true identity. To verify this, you can graph the two equations $y = \cot A$ and $y = \frac{\cos A}{\sin A}$ and verify that they give the same



$$BaCl_2(aq) + Na_2SO_4(aq) \longrightarrow BaSO_4(s) + 2\ NaCl(aq)$$

Step 3. Identify the ions in solution. All soluble ionic compounds dissociate to form ions in aqueous solution. (All are electrolytes.)

$$BaCl_2(s) \longrightarrow Ba^{2+}(aq) + 2\ Cl^-(aq)$$
$$Na_2SO_4(s) \longrightarrow 2\ Na^+(aq) + SO_4^{2-}(aq)$$